JHEP HYPER VERSION

Received: *July 12, 2001*, Accepted: *July 19, 2001*

Domain walls with strings attached

Renata Kallosh

Department of Physics, Stanford University
Stanford, CA 94305, USA
E-mail: kallosh@stanford.edu

Sergey Prokushkin

Department of Physics, Stanford University
Stanford, CA 94305, USA
E-mail: prok@stanford.edu

Marina Shmakova

CIPA, 366 Cambridge Avenue
Palo Alto, CA 94306, USA and
SLAC, Stanford University
Stanford, CA 94309, USA
E-mail: shmakova@slac.stanford.edu

Abstract: We have constructed a bulk & brane action of IIA theory which describes a pair of BPS domain walls on $S_1/\mathbb{Z}_2$, with strings attached. The walls are given by two orientifold O8-planes with coincident D8-branes and 'F1-D0'-strings are stretched between the walls. This static configuration satisfies all matching conditions for the string and domain wall sources and has 1/4 of unbroken supersymmetry.

Keywords: Extra Large Dimensions, p-branes, Classical Theories of Gravity, Supergravity Models.

JHEP07(2001)023

Contents

1. Introduction

Recently a new approach to type-IIA theory in 10d was initiated in [1]. It was given a name 'bulk & brane action'.[1] The purpose was to clarify the properties of D8 branes (domain walls in 10d) and related configurations which require the 'massive' type-IIA bulk supergravity of Romans [5]. The new version of IIA supergravity in the bulk [1] includes a 10-form field strength $G^{(10)}$ which is dual to zero-form field strength $G^{(0)}$. The theory is defined on $S_1/\mathbb{Z}_2$ space. The value of $G^{(0)}$ depends on the brane part of the bulk & brane action. The brane part consists of two orientifold planes and some number of D8 branes coincident with each of the O8 planes. The resulting *on shell value of the 0-form field $G^{(0)}$ in the presence of sources is a piecewise constant*

$$G^{(0)}(x^9) = \frac{2n-16}{4\pi l_s}\epsilon(x^9)\,.$$

This identifies the mass parameter of type-IIA supergravity in 'bulk & brane action' [1] as follows:

$$m = \begin{cases} \dfrac{n-8}{2\pi\ell_s}\,, & 0 < x^9 < \pi R\,, \\ \\ -\dfrac{n-8}{2\pi\ell_s}\,, & -\pi R < x^9 < 0\,. \end{cases} \tag{1.1}$$

The mass is quantized in string units and it is proportional to $n-8$ where there are $2n$ and $2(16-n)$ D8-branes (including the images) at each O8-plane. The mass

[1] This was a follow up of a 5d bulk & brane action suggested in [2] with the purpose to supersymmetrize the Randall-Sundrum [3] brane world. This approach to IIA theory is closely related to Polchinski-Witten construction [4].

vanishes in the special case $n = 8$ when the contribution from the D8-branes cancels exactly the contribution from the O8-planes,

$$n = 8 \qquad \Rightarrow \qquad m = 0\,.$$

The set up for the bulk & brane action allows to find the equations of motion following from the bulk supergravity as well as from the brane actions. A familiar example of this kind is the fundamental string F1: the equations of motion were derived from the bulk 10d massless supergravity supplemented by the string action [6]. The coordinates of the string were embedded into the target space of supergravity by the choice of the static gauge: $X^0 = \tau, X^1 = \sigma$. The matching conditions at the position of the string were satisfied for this solution. In [1] an analogous procedure was performed for stringy domain walls in massive IIA bulk supergravity supplemented by O8 and D8 actions. The solution satisfied matching conditions at the positions of the walls and had 1/2 of unbroken supersymmetry.

The purpose of this paper is to introduce more general 'bulk & brane actions' and study the solutions of massive IIA theory with less unbroken supersymmetry. The challenge here is that apart from D8 branes which require massive supergravity, the rest of the known D-branes are solutions of the massless supergravity. Few solutions of the massive supergravity have been found before [7, 8]. However they have been found in the theory with the constant mass, without a 10-form, whereas a consistent theory of domain walls on $S_1/\mathbb{Z}_2$ requires the presence of the 10-form dual to a 0-form and they both change the sign across the wall.

An interesting case which we will study in this paper is a configuration which one can conditionally call D8-D0-F1 solution. It is expected to define strings stretched between the walls. One can not expect a simple combination of known D8 brane, D0 brane and fundamental strings F1 solutions by the following reason:

- The status of the D0 brane in massive supergravity with everywhere constant mass is somewhat ambiguous since the 1-form can be gauged away via Higgs effect which makes the 2-form B a massive field [5]. In our theory [1], however, in presence of the 0- and 10-form fields it is impossible to gauge the RR 1-form away everywhere. In Romans theory [5] the field $B_{\mu\nu}$ appears either via the field strength $H = dB$ or in the combination $dA + mB$. Thus the transformation $\delta B = dA/m$ does not change H and absorbs the 1-form A into B. In [1] B enters in a combination $dA + G^{(0)}B$ where the 0-form $G^{(0)}$ is a function and therefore $G^{(0)}B$ can not absorb dA, in general.

- The fundamental string F1 of type-IIA massless supergravity does not solve equations of motion of the massive theory. Therefore in presence of O8-D8 walls some unusual strings may be expected.

Our interest to the problem was enhanced also by a phenomenon of *string creation when D0 particle crosses a D8 brane* [9]. The observation in these papers was of the following nature. One considers one isolated D8-brane and one assumes that on one side of the D8-brane, for example the right hand side, there is a bulk defined by massless supergravity,

$$m_{\rm rhs} = 0 \qquad \Rightarrow \qquad D0\,.$$

Therefore on the right hand side of the D8-brane the usual D0-branes are possible since they solve equations of motion of massless supergravity. On the other side of the D8-brane the bulk is assumed to have a non-vanishing mass, i.e. the bulk is defined by the massive supergravity. The usual D0-brane can not exist there without a $B_{\mu\nu}$-field. Thus a string must be created as soon as the D0-brane crosses the D8-branes and appears on the other side where

$$m_{\rm lhs} \neq 0 \qquad \Rightarrow \qquad D0 + F1\,.$$

In our case when we have two O8-D8 domain walls at the fixed points of the orientifold, the mass changes the sign across the wall, but is nowhere vanishing. Therefore we expect to find a solution which everywhere has a modified string combined with some modified charged D-particle.

2. Partially dualized action

We will start from $d = 10$ lagrangian for dual IIA which is given in eq. (2.23) of [1] with the independent fields[2]

$$\left\{e_\mu^a, B_{\mu\nu}, \phi, G^{(0)}, G^{(2)}_{\mu\nu}, G^{(4)}_{\mu_1\cdots\mu_4}, A^{(5)}_{\mu_1\cdots\mu_5}, A^{(7)}_{\mu_1\cdots\mu_7}, A^{(9)}_{\mu_1\cdots\mu_9}, \psi_\mu, \lambda\right\}. \tag{2.1}$$

The bulk action is

$$\begin{aligned}
S^{5,7,9}_{\rm bulk} = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Big\{ & e^{-2\phi}\Big[R(\omega(e)) - 4(\partial\phi)^2 + \frac{1}{2}H\cdot H - 2\partial^\mu\phi\chi^{(1)}_\mu + H\cdot\chi^{(3)} + \\
& \qquad + 2\bar\psi_\mu\Gamma^{\mu\nu\rho}\nabla_\nu\psi_\rho - 2\bar\lambda\Gamma^\mu\nabla_\mu\lambda + 4\bar\lambda\Gamma^{\mu\nu}\nabla_\mu\psi_\nu\Big] + \\
& + \sum_{n=0,1,2}\frac{1}{2}G^{(2n)}\cdot G^{(2n)} + G^{(2n)}\cdot\Psi^{(2n)} + \\
& - \star\Big[\frac{1}{2}G^{(4)}G^{(4)}B - \frac{1}{2}G^{(2)}G^{(4)}B^2 + \frac{1}{6}G^{(2)2}B^3 + \\
& \qquad + \frac{1}{6}G^{(0)}G^{(4)}B^3 - \frac{1}{8}G^{(0)}G^{(2)}B^4 + \frac{1}{40}G^{(0)2}B^5 + \\
& \qquad + \mathbf{e}^{-B}\mathbf{G}d(A^{(5)} - A^{(7)} + A^{(9)})\Big]\Big\} + \\
& + \text{quartic fermionic terms}\,,
\end{aligned} \tag{2.2}$$

[2] We keep all notation of [1].

where $\mathbf{G} = \sum_{n=0}^{5} G^{(2n)}$ is a formal sum and

$$\begin{aligned}
\chi^{(1)}_{\mu} &= -2\bar{\psi}_\nu \Gamma^\nu \psi_\mu - 2\bar{\lambda}\Gamma^\nu \Gamma_\mu \psi_\nu \,, \\
\chi^{(3)}_{\mu\nu\rho} &= \frac{1}{2}\bar{\psi}_\alpha \Gamma^{[\alpha}\Gamma_{\mu\nu\rho}\Gamma^{\beta]}\mathcal{P}\psi_\beta + \bar{\lambda}\Gamma_{\mu\nu\rho}{}^{\beta}\mathcal{P}\psi_\beta - \frac{1}{2}\bar{\lambda}\mathcal{P}\Gamma_{\mu\nu\rho}\lambda \,, \\
\Psi^{(2n)}_{\mu_1\cdots\mu_{2n}} &= \frac{1}{2}e^{-\phi}\bar{\psi}_\alpha \Gamma^{[\alpha}\Gamma_{\mu_1\cdots\mu_{2n}}\Gamma^{\beta]}\mathcal{P}_n\psi_\beta + \frac{1}{2}e^{-\phi}\bar{\lambda}\Gamma_{\mu_1\cdots\mu_{2n}}\Gamma^{\beta}\mathcal{P}_n\psi_\beta + \\
&\quad - \frac{1}{4}e^{-\phi}\bar{\lambda}\Gamma_{[\mu_1\cdots\mu_{2n-1}}\mathcal{P}_n\Gamma_{\mu_{2n}]}\lambda \,.
\end{aligned}$$

The fields $G^{(0)}, G^{(2)}_{\mu\nu}, G^{(4)}_{\mu_1\cdots\mu_4}$ are auxiliary since they enter into the action without derivatives and can be integrated out so that the action will depend on the field strengths of $A^{(5)}_{\mu_1\cdots\mu_5}, A^{(7)}_{\mu_1\cdots\mu_7}, A^{(9)}_{\mu_1\cdots\mu_9}$ R-R forms. Alternatively, it was explained in [1], one can dualize this action and bring it to the form closely related to the original action of Romans [5] where the 1-form $C^{(1)}_{\mu}$ and the 3-form $C^{(3)}_{\mu\nu\lambda}$ appear and the mass is constant. The change of the basis for RR-forms must be performed for such transition, $\mathbf{A} = \mathbf{C} \wedge e^{-B}$. The action in stringy frame is given in [1, eq. (2.33)].

None of these actions, neither the one in (2.2) nor the Romans-type action, can be used directly to find a solution we are looking for, since we need both a 1-form for D0 and a 9-form for O8-D8 wall. However, it is easy to bring the action (2.2) to a desirable form.

Our goal therefore is to construct a partially dual Lagrangian in terms of independent fields

$$\left\{e^a_\mu, B_{\mu\nu}, \phi, G^{(0)}, G^{(4)}_{\mu_1\cdots\mu_4}, A^{(1)}_\mu, A^{(5)}_{\mu_1\cdots\mu_5}, A^{(9)}_{\mu_1\cdots\mu_9}, \psi_\mu, \lambda\right\} \tag{2.3}$$

that will correspond to the D0-F1-D8 brane system. We can express the auxiliary field $G^{(2)}$ via $A^{(1)}$, $G^{(0)}$ and B using the field equations for $A^{(7)}$ following from (2.2)

$$-\int \mathbf{e}^{-B}\mathbf{G}{\scriptstyle\wedge} dA^{(7)} = \int (d\mathbf{e}^{-B}\mathbf{G}){\scriptstyle\wedge} A^{(7)} + d[\cdots] \qquad \Rightarrow \qquad d(G^{(2)} - G^{(0)}B) = 0\,.$$

The most general solution is [1]: $G^{(2)} = dA^{(1)} + G^{(0)}B + G^{(2)}_{\text{flux}}$, where $dG^{(2)}_{\text{flux}} = 0$. We will choose $G^{(2)}_{\text{flux}} = 0$ and will substitute the solution

$$G^{(2)}(A^{(1)}, G^{(0)}, B) = dA^{(1)} + G^{(0)}B \tag{2.4}$$

into (2.2) which will give us a partially dual action we are looking for :

$$\begin{aligned}
S^{1,5,9}_{\text{bulk}} = -\frac{1}{2\kappa^2_{10}} \int d^{10}x\sqrt{-g}\Big\{ e^{-2\phi}\Big[& R(\omega(e)) - 4(\partial\phi)^2 + \frac{1}{2}H\cdot H - 2\partial^\mu\phi\chi^{(1)}_\mu + H\cdot\chi^{(3)} + \\
& + 2\bar{\psi}_\mu\Gamma^{\mu\nu\rho}\nabla_\nu\psi_\rho - 2\bar{\lambda}\Gamma^\mu\nabla_\mu\lambda + 4\bar{\lambda}\Gamma^{\mu\nu}\nabla_\mu\psi_\nu \Big] + \\
& + \frac{1}{2}G^{(0)}\cdot G^{(0)} + \frac{1}{2}G^{(4)}\cdot G^{(4)} + \frac{1}{2}(dA^{(1)} + G^{(0)}B)\times
\end{aligned}$$

$$\times\,(dA^{(1)}+G^{(0)}B)+G^{(0)}\Psi^{(0)}+(dA^{(1)}+G^{(0)}B)\cdot\Psi^{(2)}+$$
$$+\,G^{(4)}\cdot\Psi^{(4)}-\star\Big[\frac{1}{2}\,G^{(4)}G^{(4)}B-\frac{1}{3}\,G^{(0)}G^{(4)}B^3+$$
$$+\,\frac{1}{15}\,G^{(0)2}B^5+\frac{1}{6}dA^{(1)}dA^{(1)}B^3+$$
$$+\left(\frac{5}{24}G^{(0)}B^4-\frac{1}{2}G^{(4)}B^2\right)dA^{(1)}+$$
$$+\left(G^{(4)}-BdA^{(1)}-\frac{1}{2}G^{(0)}B_\wedge B\right)dA^{(5)}+$$
$$+\,G^{(0)}dA^{(9)}\Big]\Big\}+$$
$$+\ \text{quartic fermionic terms}\,. \tag{2.5}$$

Substitution of (2.4) into supersymmetry transformation rules found in [1] for the action (2.2) gives the supersymmetry transformations of our new partially dual action (2.5):

$$\begin{aligned}
\delta_\epsilon\psi_\mu &= \left(\partial_\mu+\frac{1}{4}\,\not{\omega}_\mu+\frac{1}{8}\Gamma_{11}\,\not{H}_\mu\right)\epsilon+\\
&\quad+\frac{1}{8}e^{\phi}\left(G^{(0)}\Gamma_\mu+\frac{1}{2}(2\,\partial_{[\nu}A^{(1)}_{\rho]}\Gamma^{\nu\rho}+G^{(0)}\,\not{B})\Gamma_\mu\Gamma_{11}+\frac{1}{24}\,\not{G}^{(4)}\Gamma_\mu\right)\epsilon\,,\\
\delta_\epsilon\lambda &= \left(\not{\partial}\phi-\frac{1}{12}\Gamma_{11}\,\not{H}\right)\epsilon+\frac{1}{4}e^{\phi}\left(5G^{(0)}+\frac{3}{2}(2\partial_{[\nu}A^{(1)}_{\rho]}\Gamma^{\nu\rho}+G^{(0)}\,\not{B})\Gamma_{11}+\frac{1}{24}\,\not{G}^{(4)}\right)\epsilon\,,\\
\delta_\epsilon\phi &= \frac{1}{2}\,\bar\epsilon\lambda\,,\\
\delta G^{(0)} &= 0\,,\\
\delta A^{(1)} &= -e^{-\phi}\bar\epsilon\Gamma_{11}\left(\psi_\mu-\frac{1}{2}\Gamma_\mu\lambda\right)\,,\\
\delta G^{(2)} &= dE^1+G^{(0)}{}_\wedge\delta_\epsilon B=\delta(dA^{(1)}+G^{(0)}B)\,,\\
\delta G^{(4)} &= dE^3+G^{(2)}{}_\wedge\delta_\epsilon B-H_\wedge E^1\,,\\
\delta A^{(5)} &= E^5-B_\wedge E^3+\frac{1}{2}B_\wedge B_\wedge E^1\,,\\
\delta A^{(9)} &= E^9-B_\wedge E^7+\frac{1}{2}B_\wedge B_\wedge E^5-\frac{1}{6}B_\wedge B_\wedge B_\wedge E^3+\frac{1}{24}B_\wedge B_\wedge B_\wedge B_\wedge E^1\,,
\end{aligned}\tag{2.6}$$

where

$$E^{(2n-1)}_{\mu_1\cdots\mu_{2n-1}}\equiv -e^{-\phi}\,\bar\epsilon\,\Gamma_{[\mu_1\cdots\mu_{2n-2}}\,(\Gamma_{11})^n\left((2n-1)\psi_{\mu_{2n-1}]}-\frac{1}{2}\Gamma_{\mu_{2n-1}]}\lambda\right).$$

3. Walls and strings

We now make an assumption that our solution has $G^{(4)}=0$, $A^{(5)}=0$, $B\wedge B=0$, $dA^{(1)}\wedge B=0$, $dA^{(1)}\wedge dA^{(1)}=0$. The full action whose variation will define the

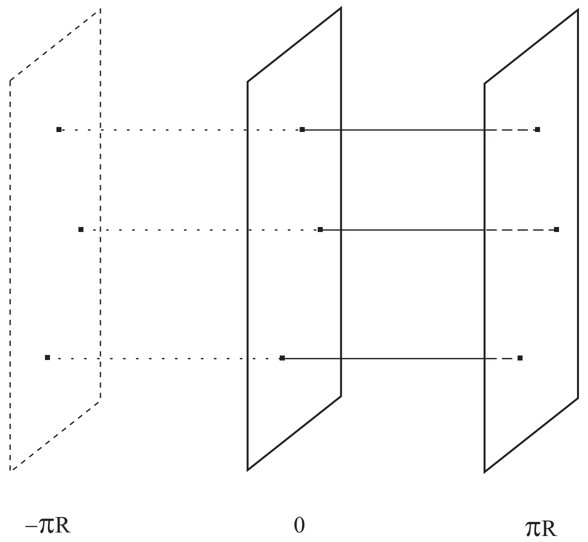

Figure 1: At $z = 0$ there is an O8-plane with $2n$ D8-branes on it, at $|z| = \pi R$ there is a second O8-plane with $2(16-n)$ D8-branes on it (counting the images). The configuration includes an orthogonal to domain walls 'F1-D0' collection of strings sitting at positions $\vec{x}_k$, all in static equilibrium, 1/4 of supersymmetry unbroken.

D0-D8-F1 solution will consist of the bulk action and source actions. The brane source action for domain walls was presented in [1]. The sources for F1-D0 are not known and we hope to find them when the bulk solution will be established. Thus we take

$$S_{\text{bulk\&brane}} = S_{\text{bulk}} + S_{O8D8} + S_{F1D0} \,. \tag{3.1}$$

The simplified form of the bulk bosonic action (2.5) which we need for our solution is:

$$\begin{aligned} S_{\text{bulk}} = -\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Big\{ & e^{-2\phi}\left[R(\omega(e)) - 4(\partial\phi)^2 + \frac{1}{12}(H_{\mu\nu\lambda})^2\right] + \frac{1}{2}(G^{(0)})^2 + \\ & + \frac{1}{2}(dA^{(1)} + G^{(0)}B)^2 - \star[G^{(0)}dA^{(9)}]\Big\} \,. \end{aligned} \tag{3.2}$$

The action of O8D8 (which is a D8-brane action in a static gauge, with excitations on the brane frozen) is

$$S_{O8+D8} = -\mu_8\int d^{10}x\,\left\{e^{-\phi}\sqrt{|g_{(9)}|} + \alpha\frac{1}{9!}\varepsilon^{(9)}A^{(9)}\right\}(\delta(z) - \delta(z-\pi R)) \;, \tag{3.3}$$

where $\mu_8 = \frac{1}{2\kappa_{10}^2}\frac{2(n-8)}{(2\pi\ell_s)}$. Here -16 is the contribution to the tension from each orientifold plane. The positive part of the tension comes from D8 branes coincident with the O-planes.

We will find the following D0-D8-F1 solution[3] on $S^1/\mathbb{Z}_2$. It depends on 2 harmonic functions, $h(z)$, where z is a coordinate transverse to the D8 brane, and $f(x^i)$ where x^i are the 8 coordinates transverse to the string.

$$\begin{aligned} ds^2 &= -h^{-1/2}f^{-3/2}dt^2 + h^{1/2}f^{-1/2}dz^2 + h^{-1/2}f^{1/2}(dx^i)^2 && (3.4)\\ e^\phi &= h^{-5/4}f^{1/4}, \qquad B_{zt} = af^{-1}, \qquad A_t = bhf^{-1} && (3.5)\\ G^{(2)}_{it} &= bh\partial_i f^{-1}, \qquad G^{(2)}_{zt} = 0 && (3.6)\\ G^{(0)} &= -\star G^{(10)} = \alpha\frac{n-8}{2\pi l_s}\epsilon(z) = -\alpha\partial_z h\,, && (3.7) \end{aligned}$$

where $\alpha^2 = a^2 = b^2 = 1$, $b = \alpha\, a$ and $G^{(2)} = dA + G^{(0)}B$ as in (2.4). Harmonic functions are defined as follows:

$$f(\vec{x}) = 1 + \sum_k \frac{N_k\, c_f^{10}}{|\vec{x}-\vec{x}_k|^6} \qquad (3.8)$$

$$h(z) = 1 - \frac{n-8}{2\pi l_s}|z|\,, \qquad \alpha^2 = 1\,, \qquad (3.9)$$

where

$$c_f^{10} = \frac{2\kappa_{10}^2}{2\pi l_s^2 6\omega_7} \qquad \omega_7 = \frac{2\pi^{7/2}}{\Gamma(7/2)}\,. \qquad (3.10)$$

The solutions of the equations of motion given above solve the equation of the action $S_{\text{bulk}} + S_{O8D8}$ everywhere but at the positions of the F1-D0 strings at $\vec{x} = \vec{x}_k$. The form of the solutions suggest that one can find the source action for F1-D0 strings so that the total action has equation of motion for which (3.4)–(3.10) gives a solution everywhere, including the position of the strings at $\vec{x} = \vec{x}_k$. We find the appropriate action in the form:

$$\begin{aligned} S_{F1D0} = -\frac{1}{2\pi\alpha'}\Bigg[\int d^{10}x \sum_k N_k\delta^8(\vec{x}-\vec{x}_k)\,\times \\ \times\left\{\left(e^{-\phi}\sqrt{-g_{tt}} - bA_t\right) + \left(\sqrt{-g_{tt}g_{zz}} - \frac{a}{2!}\epsilon^{\underline{\nu\lambda}}B_{\underline{\nu\lambda}}\right)\frac{e^{-\phi}}{\sqrt{g_{zz}}}\right\}\Bigg] \end{aligned} \qquad (3.11)$$

Here $\underline{\nu}, \underline{\lambda}$ take values t, z. As we see here, the action is not simply related to D0 or F1 solution. The first part somehow reminds a D0 action in a static gauge, however it is integrated over z and not only over t which would correspond to a D0 brane. The second part is almost an F1 string action in a static gauge. However, there is a term $\frac{e^{-\phi}}{\sqrt{g_{zz}}}$ which for our solution is equal to $h(z)$ and is related to O8-D8 domain wall. This term breaks the $O(1,1)$ symmetry of the fundamental string and is unusual.

[3] Our ansatz is motivated by the results from [8] where a closely related solution was obtained in Romans theory [5] with everywhere constant mass. On the other hand, for $G^{(0)}(x) = -\star G^{(10)}(x)$, when the string harmonic function f is trivial, $N_k = 0$ and $f = 1$, our solution is reduced to the one for two O8-D8 domain walls on $S^1/\mathbb{Z}_2$ derived in [1].

We will present below all equations of motion following from the bulk & brane action and show that they are solved by (3.4)–(3.10). Our total bulk & brane action which is a subject for variation over $\left\{g_{\mu\nu}, B_{\mu\nu}, \phi, G^{(0)}, A^{(1)}_{\mu}, A^{(9)}_{\mu_1\cdots\mu_9}\right\}$ fields is:

$$\begin{aligned}S_{\text{total}} &= S_{\text{bulk}} + S_{\text{branes}} \\ &= \frac{1}{2\kappa_{10}^2}\Big[\int d^{10}x \mathcal{L}_{\text{bulk}} - \tilde{T}_8 \int d^{10}x\, (\delta(z) - \delta(z-\pi R)) \left(e^{-\phi}\sqrt{|g_9|} + \alpha \frac{1}{9!}\epsilon^{(9)} A^{(9)}\right) - \\ &\quad - \tilde{T}_2 \int d^{10}x \sum_k N_k \delta^8(\vec{x} - \vec{x}_k)\Big\{(e^{-\phi}\sqrt{-g_{tt}} - bA_t) + \\ &\quad + \left(\sqrt{-g_{tt}g_{zz}} - \frac{a}{2!}\epsilon^{\underline{\nu\lambda}} B_{\underline{\nu\lambda}}\right)\frac{e^{-\phi}}{\sqrt{g_{zz}}}\Big\}\Big]\end{aligned} \tag{3.12}$$

The covariant equations of motion that follow from the bulk action (3.2) are:

$$\begin{aligned}\frac{\delta S}{\delta g^{\mu\nu}}|_{\text{bulk}} = &-\frac{\sqrt{-g}}{2\kappa_{10}^2}\Big(e^{-2\phi}\Big[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + 2g_{\mu\nu}\nabla^2\phi - 2\nabla_\mu\nabla_\nu\phi - \\ &-2g_{\mu\nu}\partial^\sigma\phi\partial_\sigma\phi - \frac{1}{24}g_{\mu\nu}H_{\lambda\rho\sigma}H^{\lambda\rho\sigma} + \frac{1}{4}H_{\{\mu\lambda\sigma}H^{\lambda\sigma}_{\nu\}}\Big] - \\ &- \frac{1}{4}g_{\mu\nu}(G^{(0)})^2 - \frac{1}{8}g_{\mu\nu}(G^{(2)})_{\lambda\sigma}(G^{(2)})^{\lambda\sigma} + \\ &+ \frac{1}{2}(G^{(2)})_{\{\mu\lambda}(G^{(2)})^{\lambda}_{\nu\}}\Big)\end{aligned} \tag{3.13}$$

$$\frac{\delta S}{\delta \phi}|_{\text{bulk}} = -\frac{\sqrt{-g}}{\kappa_{10}^2}e^{-2\phi}\Big(4\nabla^2\phi - 4\partial_\mu\phi\partial^\mu\phi - R - \frac{1}{12}H_{\lambda\rho\sigma}H^{\lambda\rho\sigma}\Big) \tag{3.14}$$

$$\frac{\delta S}{\delta B^{\mu\nu}}|_{\text{bulk}} = -\frac{\sqrt{-g}}{2\kappa_{10}^2}\Big(-D^\lambda(e^{-2\phi}\frac{1}{2}H_{\mu\nu\lambda}) + \frac{1}{2}G^{(0)}(G^{(2)})_{\mu\nu}\Big) \tag{3.15}$$

$$\frac{\delta S}{\delta A^{(1)}_\nu}|_{\text{bulk}} = -\frac{\sqrt{-g}}{2\kappa_{10}^2}\Big(-D_\mu(G^{(2)})^{\mu\nu}\Big) \tag{3.16}$$

$$\frac{\delta S}{\delta A^{(9)}_{\mu_1\cdots\mu_9}}|_{\text{bulk}} = -\frac{1}{2\kappa_{10}^2}\Big(-\frac{1}{9!}\epsilon^{(10)\mu_1\cdots\mu_9\nu}\partial_\nu G^{(0)}\Big) \tag{3.17}$$

$$\frac{\delta S}{\delta G^{(0)}}|_{\text{bulk}} = -\frac{\sqrt{-g}}{2\kappa_{10}^2}\Big(G^{(0)} + \frac{1}{2}(G^{(2)})_{\mu\nu}B^{\mu\nu} - \frac{1}{9!\sqrt{-g}}\epsilon^{(10)\mu_1\cdots\mu_{10}}\partial_{\mu_1}A^{(9)}_{\mu_2\cdots\mu_{10}}\Big) \tag{3.18}$$

It is easy to see that the last equation becomes a standard duality equation when $(G^{(2)})_{\mu\nu}B^{\mu\nu} = 0$, which is indeed a property of our solution (3.4)–(3.10). When our ansatz (3.4)–(3.7) is substituted in bulk equations of motion, we find a wonderful simplification

$$\frac{\delta S}{\delta g^{tt}}|_{\text{bulk}} = \frac{g_{tt}}{2\kappa_{10}^2}\Big(hf^{-1}\boldsymbol{\Delta} f + \frac{1}{2}h^{-1}f\partial_z\partial_z h\Big), \tag{3.19}$$

$$\frac{\delta S}{\delta g^{zz}}|_{\text{bulk}} = \frac{g_{zz}}{4\kappa_{10}^2}\Big(hf^{-1}\boldsymbol{\Delta} f\Big), \tag{3.20}$$

$$\frac{\delta S}{\delta g^{\mathrm{ii}}}|_{\mathrm{bulk}} = \frac{g_{\mathrm{ii}}}{4\kappa_{10}^2}\Big(h^{-1}f\partial_z\partial_z h\Big)\,, \tag{3.21}$$

$$\frac{\delta S}{\delta \phi}|_{\mathrm{bulk}} = \frac{1}{2\kappa_{10}^2}(hf^{-1}\boldsymbol{\Delta} f + h^{-1}f\partial_z\partial_z h)\,, \tag{3.22}$$

$$\frac{\delta S}{\delta B_{zt}}|_{\mathrm{bulk}} = \frac{a}{4\kappa_{10}^2}h\boldsymbol{\Delta} f\,, \tag{3.23}$$

$$\frac{\delta S}{\delta A_t^{(1)}}|_{\mathrm{bulk}} = \frac{1}{2\kappa_{10}^2}\frac{a}{\alpha}\boldsymbol{\Delta} f\,, \tag{3.24}$$

$$\frac{\delta S}{\delta A^{(9)}_{\mu_1\cdots\mu_9}}|_{\mathrm{bulk}} = -\frac{\alpha}{2\kappa_{10}^2}\frac{\epsilon^{(9)}}{9!}\Big\{\partial_z\partial_z h(z)\Big\}\,, \tag{3.25}$$

$$\frac{\delta S}{\delta G^{(0)}}|_{\mathrm{bulk}} = -\frac{\sqrt{-g}}{2\kappa_{10}^2}\Big(G^{(0)} + \star G^{(10)}\Big)\,. \tag{3.26}$$

In the presence of sources introduced in eq. (3.12) the complete equations of motion have additional terms:

$$\begin{aligned}\frac{\delta S}{\delta g^{tt}}|_{\mathrm{bulk\&brane}} = {} & \frac{g_{tt}}{2\kappa_{10}^2}\Big\{hf^{-1}\boldsymbol{\Delta} f + \tilde{T}_2 hf^{-1}\sum_k N_k\delta^8(\vec{x}-\vec{x}_k)\Big\} + \\ & +\frac{g_{tt}}{4\kappa_{10}^2}\Big\{h^{-1}f\partial_z\partial_z h + \tilde{T}_8 h^{-1}f(\delta(z)-\delta(z-\tilde{z}))\Big\} = 0\,,\end{aligned} \tag{3.27}$$

$$\frac{\delta S}{\delta g^{zz}}|_{\mathrm{bulk\&brane}} = \frac{g_{zz}}{4\kappa_{10}^2}\Big\{hf^{-1}\boldsymbol{\Delta} f + \tilde{T}_2 hf^{-1}\sum_k N_k\delta^8(\vec{x}-\vec{x}_k)\Big\} = 0\,, \tag{3.28}$$

$$\frac{\delta S}{\delta g^{\mathrm{ii}}}|_{\mathrm{bulk\&brane}} = \frac{g_{\mathrm{ii}}}{4\kappa_{10}^2}\Big\{h^{-1}f\partial_z\partial_z h + \tilde{T}_8 h^{-1}f(\delta(z)-\delta(z-\tilde{z}))\Big\} = 0\,, \tag{3.29}$$

$$\begin{aligned}\frac{\delta S}{\delta \phi}|_{\mathrm{bulk\&brane}} = {} & \frac{1}{2\kappa_{10}^2}\Big\{hf^{-1}\boldsymbol{\Delta} f + \tilde{T}_2 hf^{-1}\sum_k N_k\delta^8(\vec{x}-\vec{x}_k)\Big\} + \\ & +\frac{1}{2\kappa_{10}^2}\Big\{h^{-1}f\partial_z\partial_z h + \tilde{T}_8 h^{-1}f(\delta(z)-\delta(z-\tilde{z})) = 0\,,\end{aligned} \tag{3.30}$$

$$\frac{\delta S}{\delta B_{zt}}|_{\mathrm{bulk\&brane}} = \frac{a}{4\kappa_{10}^2}\Big\{h\boldsymbol{\Delta} f + \tilde{T}_2 h\sum_k N_k\delta^8(\vec{x}-\vec{x}_k)\Big\} = 0\,, \tag{3.31}$$

$$\frac{\delta S}{\delta A_t^{(1)}}|_{\mathrm{bulk\&brane}} = \frac{1}{2\kappa_{10}^2}\frac{a}{\alpha}\Big\{\boldsymbol{\Delta} f + \tilde{T}_2\sum_k N_k\delta^8(\vec{x}-\vec{x}_k)\Big\} = 0\,, \tag{3.32}$$

$$\frac{\delta S}{\delta A^{(9)}_{\mu_1\cdots\mu_9}}|_{\mathrm{bulk\&brane}} = -\frac{\alpha}{2\kappa_{10}^2}\frac{\epsilon^{(9)}}{9!}\Big\{\partial_z\partial_z h + \tilde{T}_8(\delta(z)-\delta(z-\tilde{z}))\Big\} = 0\,, \tag{3.33}$$

$$\frac{\delta S}{\delta G^{(0)}}|_{\mathrm{bulk\&brane}} = -\frac{\sqrt{-g}}{2\kappa_{10}^2}\Big(G^{(0)} + \star G^{(10)}\Big) = 0\,. \tag{3.34}$$

All equations of motion of bulk & brane action are miraculously satisfied under condition that the equations for the harmonic functions f, h include the domain wall and multi-string source terms:

$$\boldsymbol{\Delta} f + \tilde{T}_2\sum_k N_k\delta^8(\vec{x}-\vec{x}_k) = 0 \tag{3.35}$$

$$\partial_z\partial_z h + \tilde{T}_8(\delta(z)-\delta(z-\pi R)) = 0\,. \tag{3.36}$$

These equations are satisfied by our harmonic functions defined in (3.8)–(3.10).

In conclusion, we have found a 1/4 BPS solution of IIA (massive) theory with domain walls at the fixed points of the orientifold and multiple strings stretched between domain walls. The configuration has some electric field, $A_t(z, \vec{x})$ reminiscent of the D0-brane and some 2-form $B_{zt}(\vec{x})$ reminiscent of the F1 multi-string solution. There is a piecewise constant 0-form, dual to a 10-form, and both change the sign across the wall. We leave it to future investigations to find a better interpretation of this configuration and to understand the possibilities to use it.

Acknowledgments

We had useful discussions with O. Bergman, E. Halyo, S. Hellerman, I. Klebanov, A. Linde, J. McGreevy, A. Peet and J. Polchinski. We are grateful to Y. Zunger for his GRONK program[4] which we used for calculations. The work is supported by NSF grant PHY-9870115. The work of M.S. was partly supported by the Department of Energy under a contract DE-AC03-76SF00515. S.P. acknowledges the support from Stanford Graduate Fellowship Foundation.

A. 1/4 of unbroken supersymmetry

Our solution has an SO(8) symmetry. For simplicity we will switch to polar coordinates and consider one string solution at $(\vec{x})^2 = r^2 = 0$. We will find that $\delta\lambda = 0$ and $\delta\psi_t = \delta\psi_z = \delta\psi_r = 0$. Due to SO(8) symmetry the variation of the gravitino $\delta\psi_i$ can be written as $\delta\psi_i = \delta_{ij} x^j \Psi$. The relation to $\delta\psi_r$ is

$$\delta\psi_r = \delta\psi_i \frac{x^i}{r} = \frac{(x^i)^2}{r}\Psi = \Psi\,. \tag{A.1}$$

Thus if we establish that $\delta\psi_r = 0$ it will follow that $\delta\psi_i = \delta_{ij} x^j \Psi = 0$.

We will substitute our solution into the supersymmetry transformations for λ:

$$\delta_\epsilon \lambda = \left(\not{\partial}\phi - \frac{1}{12}\Gamma_{11}\not{H}\right)\epsilon + \frac{1}{4}e^{\phi}\left(5G^{(0)} + \frac{3}{2}(2\,\partial_{[\nu}A^{(1)}_{\rho]}\Gamma^{\nu\rho} + G^{(0)}\not{B})\Gamma_{11}\right)\epsilon \tag{A.2}$$

and use for our ansatz that

$$\begin{aligned}
\not{\partial}\phi &= \Gamma^\mu\partial_\mu\phi = \frac{1}{4}h^{1/4}f^{-5/4}\partial_r f\Gamma^r - \frac{5}{4}h^{-5/4}f^{1/4}\partial_z h\Gamma^{\underline{z}} \\
-\frac{1}{12}\Gamma_{11}\not{H} &= \frac{\alpha - a}{2}h^{1/4}f^{-5/4}\partial_r f - \Gamma^{rzt}\Gamma^{11} \\
\frac{5}{4}e^{\phi}G^{(0)} &= -\frac{5}{4\alpha}h^{-5/4}f^{1/4}\partial_z h \\
\frac{3}{4}e^{\phi}G^{(2)}_{rt}\Gamma^{rt}\Gamma^{11} &= -\frac{3a}{4}h^{1/4}f^{-5/4}\partial_r f\Gamma^{rt}\Gamma^{11}.
\end{aligned}$$

[4] `http://itp.stanford.edu/~zunger/`

Collecting all terms we have:

$$\delta_\epsilon \lambda = \frac{1}{4} h^{1/4} f^{-5/4} \partial_r f \Gamma^r (1 - 2a\alpha \Gamma^{zt}\Gamma^{11} - 3a\Gamma^t\Gamma^{11})\epsilon - \frac{5}{4\alpha} h^{-5/4} f^{1/4} \partial_z h (1 + \alpha\Gamma^z)\epsilon$$
$$= 0\,. \tag{A.3}$$

We impose the following projectors:

$$\begin{aligned} \alpha\Gamma^z\epsilon &= -\epsilon \\ \alpha\Gamma^{zt}\Gamma^{11}\epsilon &= \beta\epsilon\,, \quad \beta^2 = 1 \\ \Gamma^t\Gamma^{11}\epsilon &= \gamma\epsilon\,, \quad \gamma^2 = 1\,. \end{aligned} \tag{A.4}$$

The first one is for D8, the second one is related to F1 and the third one (a product of the first two) is related to D0. Thus we have 1/4 of supersymmetry unbroken, since only two projectors are independent.

The dilatino transformation (A.3) vanishes under conditions that $a(2\alpha\beta + 3\gamma) = 1$. Therefore there are two possibilities:

$$\begin{aligned} &(\mathbf{1}) \quad \alpha\beta = 1\,(\alpha = \pm 1, \beta = \pm 1)\,, \quad \gamma = -1 \;\Rightarrow\; a = -1 \\ &(\mathbf{2}) \quad \alpha\beta = -1\,(\alpha = \mp 1, \beta = \pm 1)\,, \quad \gamma = 1 \;\Rightarrow\; a = 1 \end{aligned}$$

and it is easy to check the compatibility of these projectors.

In our next step we have to consider the supersymmetry transformations for ψ

$$\delta_\epsilon \psi_\mu = \Big(\partial_\mu + \frac{1}{4}\,\not{\omega}_\mu + \frac{1}{8}\Gamma_{11}\,\not{H}_\mu\Big)\epsilon + \frac{1}{8} e^\phi \Big(G^{(0)}\Gamma_\mu + \frac{1}{2}(2\,\partial_{[\nu}A^{(1)}_{\rho]}\Gamma^{\nu\rho} + G^{(0)}\,\not{B})\Gamma_\mu\Gamma_{11}\Big)\epsilon \tag{A.5}$$

and we assume that $\epsilon = (-g_{tt})^{1/4}\epsilon_0$. (In polar coordinates ϵ_0 depends on angles). The set of useful expressions is:

$$\begin{aligned} e^t_{\underline{t}} &= h^{-1/4} f^{-3/4}\,, \qquad & \not{\omega}_t &= -\frac{3}{2} f^{-2}\partial_r f \Gamma^{rt} - \frac{1}{2} h^{-3/2} f^{-1/2} \partial_z h \Gamma^{zt} \\ e^z_{\underline{z}} &= h^{1/4} f^{-1/4}\,, & \not{\omega}_z &= \frac{1}{2} h^{1/2} f^{-3/2} \partial_r f \Gamma^{rz} \\ e^r_{\underline{r}} &= h^{-1/4} f^{1/4}\,, & \not{\omega}_r &= \frac{1}{2} h^{-3/2} f^{1/2} \partial_z h \Gamma^{zr} \end{aligned}$$

Collecting all terms we get:

$$\begin{aligned} \delta_\epsilon\psi_t &= -\frac{1}{8} f^{-2}\partial_r f \Gamma^r \Big(3\Gamma^t + 2\alpha a \Gamma^z\Gamma^{11} + a\Gamma^{11}\Big)\epsilon + \frac{1}{8\alpha} h^{-3/2} f^{-1/2}\partial_z h \Gamma^t \Big(1 + \alpha\Gamma^z\Big)\epsilon \\ \delta_\epsilon\psi_z &= \frac{1}{8} h^{1/2} f^{-3/2}\partial_r f \Gamma^r \Big(\Gamma^z + 2\alpha a\Gamma^t\Gamma^{11} + a\Gamma^{zt}\Gamma^{11}\Big)\epsilon - \frac{1}{8\alpha} h^{-1}\partial_z h \Big(\alpha + \Gamma^z\Big)\epsilon \\ \delta_\epsilon\psi_r &= -\frac{1}{8} f^{-1}\partial_r f \Big(3 + 2\alpha a \Gamma^{zt}\Gamma^{11} - a\Gamma^t\Gamma^{11}\Big)\epsilon - \frac{1}{8\alpha} h^{-3/2} f^{1/2}\partial_z h \Gamma^r \Big(1 + \alpha\Gamma^z\Big)\epsilon\,. \end{aligned}$$

It is easy to show that if the spinors ϵ satisfy the projector conditions (A.4)

$$\delta\psi_t = \delta\psi_z = \delta\psi_r = 0\,.$$

As we explained above, it also means that

$$\delta\psi_i = 0 \qquad i = 1, \ldots, 8\,.$$

We have shown that our solution (3.4)–(3.7) satisfies the condition of 1/4 of unbroken supersymmetry.

References

[1] E. Bergshoeff, R. Kallosh, T. Ortin, D. Roest and A. Van Proeyen, *New formulations of $D = 10$ supersymmetry and D8–O8 domain walls*, hep-th/0103233.

[2] E. Bergshoeff, R. Kallosh and A. Van Proeyen, *Supersymmetry in singular spaces*, *J. High Energy Phys.* **10** (2000) 033 [hep-th/0007044].

[3] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221]; *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[4] J. Polchinski and E. Witten, *Evidence for heterotic - type I string duality*, *Nucl. Phys.* **B 460** (1996) 525 [hep-th/9510169].

[5] L.J. Romans, *Massive $N = 2a$ supergravity in ten-dimensions*, *Phys. Lett.* **B 169** (1986) 374.

[6] A. Dabholkar, G. Gibbons, J.A. Harvey and F. Ruiz Ruiz, *Superstrings and solitons*, *Nucl. Phys.* **B 340** (1990) 33.

[7] B. Janssen, P. Meessen and T. Ortin, *The D8-brane tied up: string and brane solutions in massive type IIA supergravity*, *Phys. Lett.* **B 453** (1999) 229 [hep-th/9901078]; Y. Imamura, *1/4 BPS solutions in massive IIA supergravity*, hep-th/0105263.

[8] M. Massar and J. Troost, *D0-D8-F1 in massive IIA SUGRA*, *Phys. Lett.* **B 458** (1999) 283 [hep-th/9901136].

[9] C.P. Bachas, M.R. Douglas and M.B. Green, *Anomalous creation of branes*, *J. High Energy Phys.* **07** (1997) 002 [hep-th/9705074];
U. Danielsson, G. Ferretti and I.R. Klebanov, *Creation of fundamental strings by crossing D-branes*, *Phys. Rev. Lett.* **79** (1997) 1984 [hep-th/9705084];
O. Bergman, M.R. Gaberdiel and G. Lifschytz, *Branes, orientifolds and the creation of elementary strings*, *Nucl. Phys.* **B 509** (1998) 194 [hep-th/9705130].